# Finding the Price to Maximize Revenue

Ted Mitchell


## Sales Revenue

R = PQ

where

R = Sales Revenue

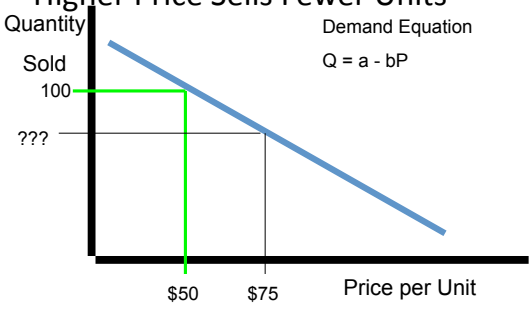
P = Price per Unit

Q = Quantity Sold (Demanded)

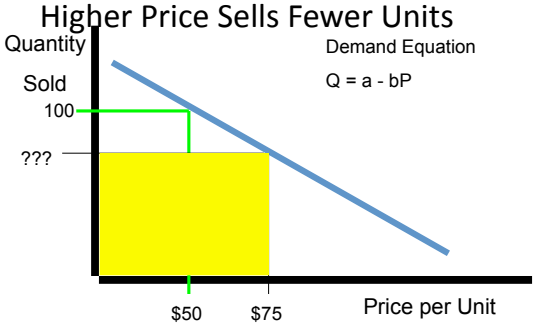
## Demand Function

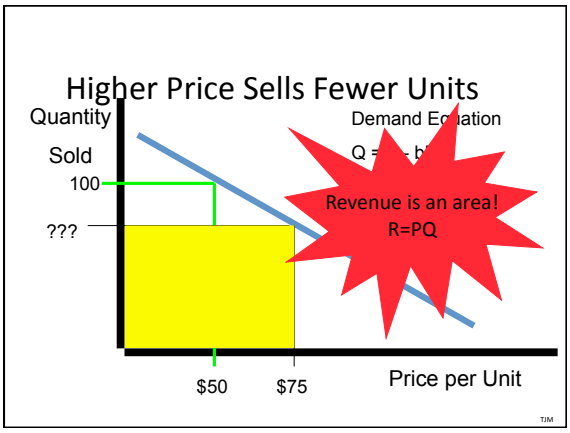
- The Quantity that will be sold is also determined by the price per unit
  Q = $f$(P)
- R = P$f$(P)

## Higher Price Sells Fewer Units




## Higher Price Sells Fewer Units






How To Calculate Demand?

Demand equation is represented by

$Q = a - bP$

Market Research provides estimates of the a & b

How To Calculate Demand?

Demand equation is represented by

$Q = a - bP$

Q = Quantity That Will Be Demanded At A Given Price

How To Calculate Demand?

Demand equation is represented by

$Q = a - bP$

a = quantity that could be given away at a zero price

How To Calculate Demand?

Demand equation is represented by

$Q = a - bP$

b = number of units or lost sales if the price is increased by one unit

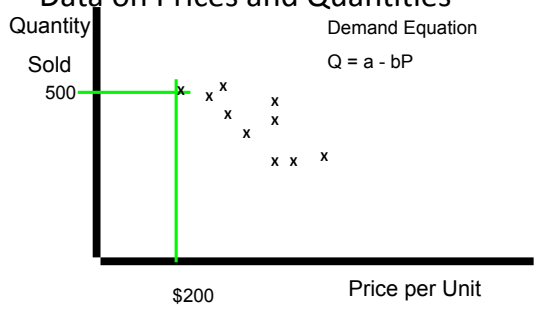
How To Calculate Demand?

Demand equation is represented by

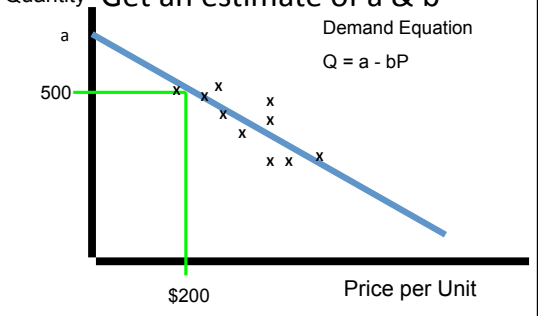
$Q = a - bP$

P = Price Per Unit

## Data on Prices and Quantities




## Get an estimate of a & b



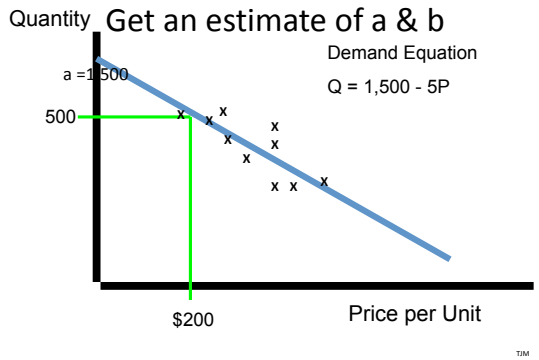

## Example

- Market research has estimated a = 1,500 and b = 5, your demand function is estimated to be Q = 1500 -5P
  The current price is $200 per unit.
- What quantity is being demanded?

## Example

- Market research has estimated your demand to be Q = 1500 -5P
  The current price is $200 per unit.
- What quantity is being Demanded?
- Q = 1500 -5(200)
- Q = 1500 -1000 = 500 units

## Get an estimate of a & b



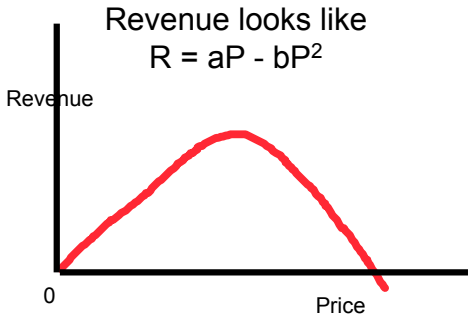

## The Revenue Equation

– Revenue, R = P xQ
  - Q = a - bP

Substitute Q = (a-bP)

- R = P(a - bP)
- $R = aP - bP^2$

## Revenue looks like $R = aP - bP^2$

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## $6 & $4 Examples

- The Demand For Your Product Changes with The Price You Charge As
  - »Q = 5000 - 500P
  - » R = PQ
  - » R = P(5000 - 500P)

## $6 Example

- R = PQ
- R = P(5000 - 500P)
- What is your total sales revenue if your price is six dollars?

## $6 Example

- R = PQ
- R = P(5000 - 500P)
  - –substitute

## $6 Example

- R = PQ
- R = P(5000 - 500P)
  - –Substitute P = 6
- R = 6(5000 - 500(6))
- R =12,000

## $4 Example

- R = PQ
- R = P(5000 - 500P)
- What is your total sales revenue if your price is four dollars?

## $4 Example

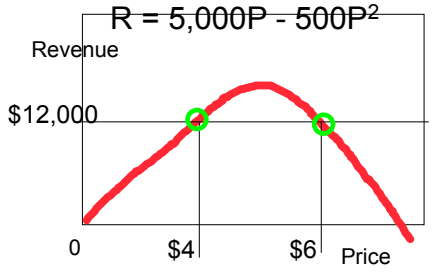
- R = PQ
- R = P(5000 - 500P)
  - –Substitute 4 = P

## $4 Example

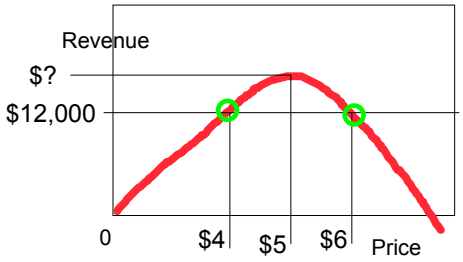
- R = PQ
- R = P(5000 - 500P)
  - –Substitute 4 = P
- R = 4(5000 - 500(4))
- R = 12,000

## How Can The $12,000 Revenue At $6 Be The Same As The Revenue At $4?

## Because Revenue looks like $R = 5{,}000P - 500P^2$




## What Happens At $5.00 $R = 5{,}000P - 500P^2$




## $5 Example ***

- R = PQ
- R = P(5000 - 500P)
- What is your total sales revenue if your price is five dollars?

## $5 Example

- R = PQ
- R = P(5000 - 500P)
  - –Substitute P=5
- R = 5(5000 - 500(5))
- R = 12,500

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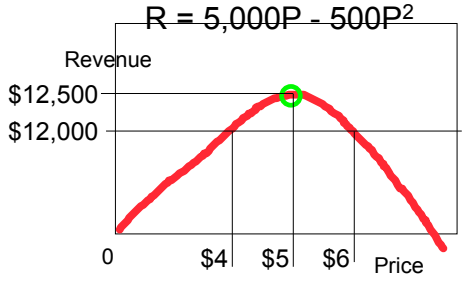
## $5 Price Maximizes Revenue???

$R = 5{,}000P - 500P^2$

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## Find the optimal Price that Maximizes Revenue

- Establish the Demand Function, Q = 5000 -500P
- Establish the Revenue function
- $R = P \times Q = P \times (5000 - 500P) = 5000P - 500P^2$
- Find the first derivative wrt price
- $dR/dP = 5000 – 2(500)P = 5000 - 1000P$
- Set the first derivative equal to zero and solve for P
- $5000 – 1000P = 0$
- $P = 5000/1000 = \$5$

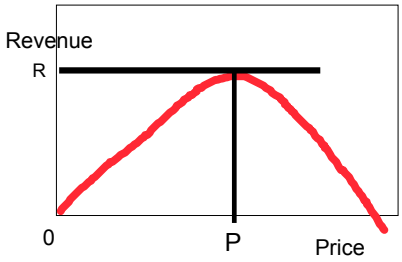
## The maximum revenue is

- Revenue = P x Q = P x (5,000 – 500P)
- Substitute optimal P = $5
- Max revenue = $5{,}000P – 500P^2$
- Max revenue = $5(5{,}000 – 500(5^2))$
- Max Revenue = 25,000 – 12,500 = $12,500

- Fortunately there is a general solution!

## Revenue looks like

$R = aP - bP^2$

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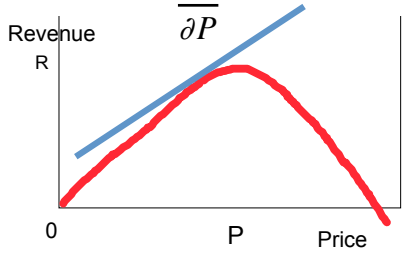
Slope of Revenue Curve is $\frac{\partial R}{\partial P}$

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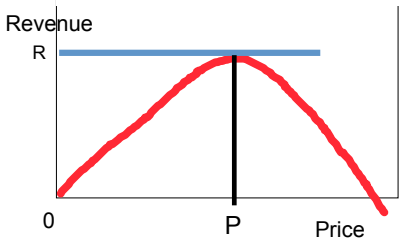
Slope Of Revenue Curve Is Zero

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Find The First Derivative of The Revenue Curve and Set It Equal to Zero

$R = aP - bP^2$

Revenue Equation

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Find The First Derivative of The Revenue Curve and Set It Equal to Zero

$R = aP - bP^2$

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Find The First Derivative of The Revenue Curve and Set It Equal to Zero

$R = aP - bP^2$

$$\frac{\partial R}{\partial P} = a - 2bP$$

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Find The First Derivative
of The Revenue Curve
and Set It Equal to Zero

$R = aP - bP^2$

Set =0

$$\frac{\partial R}{\partial P} = a - 2bP$$

$$a - 2bP = 0$$

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$$a - 2bP = 0$$

Solve for P

$$a - 2bP = 0$$

Solve for P

$$P = \frac{a}{2b}$$

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- The Demand For Your Product Changes with The Price You Charge As
  - » Q = 5000 - 500P

$$P = \frac{a}{2b}$$

$$P = \frac{5000}{2(500)} = 5$$

<u>Rule of Thumb</u>
The Price That Maximizes Revenue Is
Give-Away-Quantity
Divided by Twice The Number Of Lost
Sales For Any Dollar Increase In Price

$$P = \frac{a}{2b}$$

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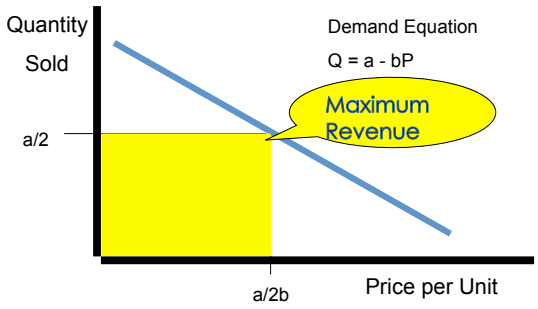
## Exam Question

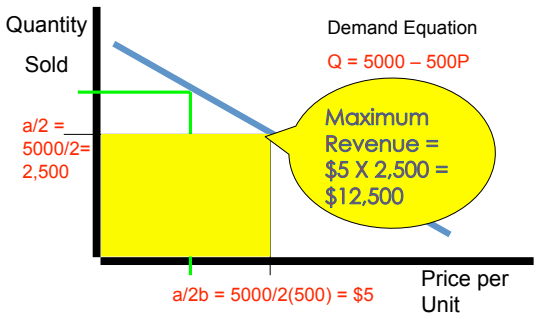
- What Is The Maximum Revenue That Can Be Generated If The Demand For The Product Is

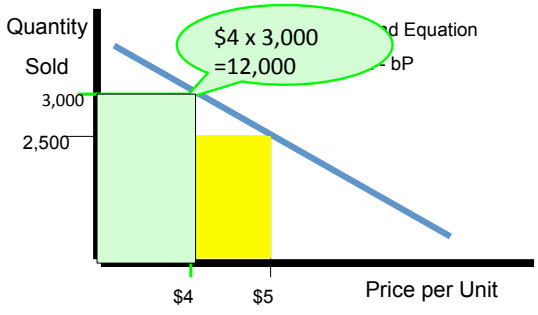
»Q = a - bP

## Optimal Price Max Rev

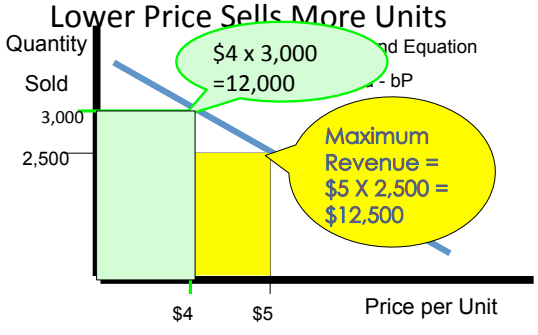



## Optimal price Max Rev




## Lower Price Sells More Units




## What we are learning?

- There is a optimal price, Pr* that maximizes sales revenue
- Too High a selling price reduces revenue
- Too Low a selling price reduces revenue